

Testing Λ CDM Cosmology with the Radiation Temperature-Redshift Relation

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Abstract

The standard Λ CDM model can be mimicked at the background and perturbative levels (linear and non-linear) by a class of gravitationally induced particle production cosmology dubbed CCDM cosmology. However, the radiation component in the CCDM model follows a slightly different temperature-redshift $T(z)$ -law which depends on an extra parameter, ν_r , describing the subdominant photon production rate. Here we perform a statistical analysis based on a compilation of 36 recent measurements of $T(z)$ at low and intermediate redshifts. The likelihood of the production rate in CCDM cosmologies is constrained by $\nu_r = 0.023 \pm 0.027$ (1σ confidence level, thereby showing that Λ CDM ($\nu_r = 0$) is still compatible with the adopted data sample. Although being hardly differentiated in the dynamic sector (cosmic history and matter fluctuations), the so-called thermal sector (temperature law, abundances of thermal relics and CMB power spectrum) offers a clear possibility for crucial tests confronting Λ CDM and CCDM cosmologies.

PACS numbers:

Keywords:

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I. INTRODUCTION

It is widely believed that the present accelerating phase of the Universe is fueled by the vacuum energy density or cosmological constant present in the the top ten Λ CDM cosmology. However, the tiny value of the vacuum energy density associated with Λ ($\rho_v = \Lambda/8\pi G$) is plagued with the cosmic coincidence and cosmological constant problems [1–3]. Such mysteries have inspired many cosmologists to propose alternative models based on rather different approaches [4], among them: modified gravity [5], quintessence [6–8], time-evolving Λ models [9–11] and gravitationally induced particle production or CCDM cosmology [12–20].

More recently, beyond these fundamental problems, it has also been observed that the predictions of the Hubble constant H_0 and matter energy density Ω_{m0} based on CMB experiments and the fitting Λ CDM cosmology are in conflict with independent measurements [21]. In particular, the tension between local determinations of H_0 based on Supernovae and other local calibrators of the cosmic distance ladder with the high redshift CMB prediction has been discussed by many authors [22, 23]. In this line, Riess and coworkers [24] reported a new measurement of the Hubble constant $H_0 = 73.02 \pm 1.79 \text{ km.s}^{-1}.\text{Mpc}^{-1}$ with an uncertainty of only 2.4%, a value 3.0σ higher than the corresponding value of $H_0 = 67.27 \pm 0.66 \text{ km.s}^{-1}.\text{Mpc}^{-1}$ predicted by the Planck experiment[21]. Such a discrepancy means that the present expansion rate can be nearly 8% faster than the value obtained from CMB data analysis by the Planck group. Perhaps more important, the tension remains to some degree even including data from different phenomena to constrain the Λ CDM model. In principle, such a discrepancy may be explained by unknown sources of systematics as argued by Planck team, however, in general grounds, problems with the fitting Λ CDM model cannot be a priori discarded. Naturally, such observational results reinforce the interest to explore new avenues including possible cosmological origins for the discrepancy, and, as such, may also indicate new clues to the unknown cosmic dark sector.

On the other hand, it has been shown that a cosmology driven by gravitationally induced particle production of all non-relativistic species existing in the present Universe mimics exactly the observed accelerating Λ CDM cosmology with just one dynamical free parameter

[20] (see also [12] for creation of cold dark matter alone). This extended scenario also provides a natural reduction of the dark sector since the vacuum component is not needed to accelerate the Universe. The late time acceleration phase is obtained with just one free parameter describing the effective production rate of the nonrelativistic components. The remarkable point here is that such a cosmic scenario is fully degenerated with the Λ CDM model at the background, as well as when matter fluctuations are taken into account both in the linear and nonlinear levels (see also [18, 19]). Different from the original CCDM cosmology [12], the associated creation process is also in agreement with the universality of the gravitational interaction in the sense that all nonrelativistic components are created and described by the specific creation rates. Tests involving only the cosmic history or fluctuations of the matter component cannot break the degeneracy presented by the Λ CDM and CCDM cosmologies.

Naturally, the mentioned universality implies that subdominant relativistic components (and relic neutrinos) are also “adiabatically” created by the evolving gravitational background. This means that probes related to the physics of the thermodynamic sector cannot be discarded as a crucial test in breaking the degeneracy between Λ CDM and CCDM models both in the non-perturbative and perturbative levels. In particular, even considering that a blackbody spectrum with temperature $T_0 = 2.725 \pm 0.002$ is currently observed [25], the cosmic thermometers providing the temperature-law $T(z)$ must be slightly affected by the subdominant “adiabatic” creation of thermalized photons, as suggested long ago [26–28] and more recently by the authors of references [12, 20] in their formulation of the CCDM cosmology (see also [29]).

In this context, by using a compilation of 36 $T(z)$ data at low and moderate redshifts, we confront here the predictions of Λ CDM and CCDM cosmologies. The former predicts the standard linear $T(z)$ law while in the latter, the temperature evolution is driven by a nonlinear law depending only on a single extra parameter which is constrained by these data over the interval $\nu_r = 0.023 \pm 0.027$ (1σ confidence level). As we shall see, although being hardly differentiated in the dynamic sector (cosmic history and matter fluctuations), cosmic probes emerging from the thermal sector like the temperature-law, abundances of thermal relics and CMB power spectrum, offer a clear possibility for crucial tests involving Λ CDM

and Λ CDM cosmologies.

II. CCDM COSMOLOGY

In this section, the formulation of the extended CCDM cosmology will be briefly reviewed. The reader interested in more physical details may consult recent articles [12–20] and also the first papers on the subject [30–33].

To begin with, let us consider a spatially flat FRW geometry whose time-evolving gravitational field is also a source of massive and massless particles. In the one-particle approach, the back reaction of the produced particles on the geometry is described by a creation pressure [30–33]. Here we consider a mixture as discussed in the extended approach where the Einstein field equations take the following form [20]:

$$8\pi G \sum_{i=1}^N \rho_i = 3H^2, \quad (1)$$

$$8\pi G \sum_{i=1}^N (p_i + P_{ci}) = -2\dot{H} - 3H^2, \quad (2)$$

where $H = \dot{a}/a$ is the Hubble parameter ($a(t)$ is the scale factor), an over-dot means time derivative with respect to the cosmic time and $\rho_i, i = 1, 2, 3 \dots N$, denotes the energy density of a given component. In order to simplify matters we consider here only baryons, cold dark matter (CDM) and the subdominant relic radiation. The quantity p_i denotes the equilibrium pressure defined by the usual equation of state (EoS)

$$p_i = \omega_i \rho_i, \quad (\omega_i = \text{const} \geq 0), \quad (3)$$

while the negative creation pressure reads [30, 32, 33]

$$P_{ci} \equiv -(\rho_i + p_i) \frac{\Gamma_i}{3H} = -(1 + \omega_i) \frac{\rho_i \Gamma_i}{3H}, \quad (4)$$

where Γ_i is the creation rate of each component and, for the last equality above, we have adopted Eq. (3). In order to define a specific matter creation scenario, a definite expression for Γ_i need to be assumed. As shown in Ref. [20], the one leading to a matter production model analogous to Λ CDM cosmology is defined by:

$$\frac{\Gamma_i}{3H} = \alpha_i \frac{\rho_{c0}}{\rho_i}. \quad (5)$$

where α_i and ρ_{c0} are, respectively, the creation rate of the i -th component and the present day value of the critical density. The former should be determined by a more fundamental theory. Note also that the presence of ω_i shows that P_{ci} as given by (4) also depends on the nature of the created components. For the 3 basic components assumed here (baryons, cold dark matter and radiation) we have, respectively, the following negative creation pressures:

$$P_{cb} = -\alpha_b \rho_{c0}, \quad P_{cdm} = -\alpha_{dm} \rho_{c0}, \quad P_{cr} = -\frac{4}{3} \alpha_r \rho_{c0}. \quad (6)$$

Note that each negative and constant creation pressure is determined by an associated free parameter. All components are decoupled and the energy conservation law for each component reads:

$$\dot{\rho}_i + 3H(\rho_i + p_i + P_{ci}) = 0, \quad (7)$$

and the total energy conservation law is contained in the field equations as should be expected (see [20] for details). The solution of the above equation is given by:

$$\rho_i = (\rho_{i0} - \alpha_i \rho_{c0}) a^{-3(1+\omega_i)} + \alpha_i \rho_{c0}, \quad (8)$$

where ρ_{i0} is the present day energy density of the i -th component.

Now, by neglecting the subdominant radiation contribution, it is readily seen that the total energy density driving the cosmic dynamics reads:

$$\rho_T = \rho_{cdm} + \rho_b = \rho_{c0} [(1 - \alpha) a^{-3} + \alpha], \quad (9)$$

where $\alpha = \alpha_{dm} + \alpha_b$. Note also that the Hubble parameter assumes the form:

$$H^2 = H_0^2 [(1 - \alpha)(1 + z)^3 + \alpha]. \quad (10)$$

The above equations (9)-(10) mimic exactly the Λ CDM expressions when the total clustering nonrelativistic matter density is identified by $\Omega_{eff} = 1 - \alpha$. It is also worth noticing that the cosmic history and the perturbative expressions of the late CCDM cosmologies

are sensitive only to the effective free parameter α describing the total creation rate. This means that the Λ CDM cosmology with creation of all nonrelativistic components emulates the Λ CDM model with just one free parameter [20].

III. CMB TEMPERATURE-LAW IN CCDM COSMOLOGY

In this section we discuss a basic question of this paper, namely: How does the CMB temperature increase when we look back in time assuming the adopted CCDM cosmology? Such a question can be answered by using two different methods. The first approach is based on the nonequilibrium thermodynamics [32, 33] while the second one (providing the same answer) comes from the associated kinetic formulation recently discussed [34]. In order to emphasize the basic assumptions we next outline the thermodynamic derivation. Its kinetic counterpart is shortly presented in the appendix A.

A. Temperature-Law and Thermodynamics

The emergence of photons into the spacetime means that the balance equilibrium equations are modified. In particular, the particle flux ($N_r^\mu{}_{;\mu} = n_r \Gamma_r$) and entropy flux ($S_r^\mu{}_{;\mu} = s_r \Gamma_r$) satisfy, respectively, equations (A) and (B) below [34]:

$$(A) \quad \dot{n}_r + 3H n_r = n_r \Gamma_r, \quad (B) \quad \dot{s}_r + 3H s_r = s_r \Gamma_r. \quad (11)$$

where n_r , s_r are, respectively, the concentration and the entropy density. We also recall that the Gibbs law reads [35]:

$$n_r T_r d\sigma_r = d\rho - \frac{\rho_r + p_r}{n_r} dn_r, \quad (12)$$

where $\sigma_r = s_r/n_r \equiv S_r/N_r$ is the specific entropy (per photon) [S_r and N_r are the entropy and number of photons per comoving volume]. Next, by taking the pair (T_r, n_r) as independent thermodynamic variables, the above equation leads to:

$$\frac{\dot{T}_r}{T_r} = \frac{1}{3} \frac{\dot{n}_r}{n_r} + \frac{\dot{\sigma}_r}{(\frac{\partial \rho_r}{\partial n_r})_{T_r}}. \quad (13)$$

By assuming that the particles are generated by the evolving Universe in such a way that the specific entropy is constant (“adiabatic” creation) it follows that:

$$\dot{\sigma}_r = 0 \Leftrightarrow \frac{\dot{S}_r}{S_r} = \frac{\dot{N}_r}{N_r} = \Gamma_r, \quad (14)$$

where in the last equality above the balance equations presented in (11) were used. This condition means that $S_r = k_B N_r$ so that the irreversibilities leading to the entropy growth is due to the emergence of particles in the spacetime. By adopting the “adiabatic” constraint and using again the equation for the particle concentration, the temperature law takes the general form for an arbitrary Γ_r expression:

$$\frac{\dot{T}_r}{T_r} = -\frac{\dot{a}}{a} + \frac{\Gamma_r}{3}. \quad (15)$$

In terms of the redshift parameter ($z = 1/a - 1$), the above equation is immediately integrated for an arbitrary creation rate [34]:

$$T_r = T_0(1+z)e^{-\int_0^z \frac{\Gamma_r}{3H} \frac{dz'}{(1+z')}}, \quad (16)$$

where T_0 is the temperature today. “Adiabatic” creation models are defined by specific creation rates Γ_r . In the case of CCDM cosmologies, it is given by Eq. (5). Using the expression for the energy density [see Eq. (8)], it is easy to show that:

$$\frac{\Gamma_r}{3H} = \alpha_r \frac{\rho_{c0}}{\rho_r} = \frac{\alpha_r}{\alpha_r + (\Omega_{r0} - \alpha_r)(1+z)^4}, \quad (17)$$

where $\Omega_{r0} = \rho_{r0}/\rho_{c0}$ is the present day radiation density parameter. By inserting this expression into (16) a simple integration yields:

$$T_r = T_0(1+z) \left[1 + \nu_r \left(\frac{1}{(1+z)^4} - 1 \right) \right]^{1/4}, \quad (18)$$

where $\nu_r = \alpha_r/\Omega_{r0}$. Note that for $\nu_r = 0$ the radiation entropy is conserved and the standard linear temperature evolution, as predicted by the Λ CDM cosmology is recovered.

The above temperature $T(z)$ law for CCDM cosmologies must be compared with the existing observations in order to constrain the free parameter ν_r or, equivalently, $\alpha_r = \nu_r \Omega_{r0}$. This is the main aim of next section.

IV. DATA SAMPLE AND STATISTICAL ANALYSIS

In Table I, we display the CMB $T(z)$ temperature data adopted here as provided by several authors in the literature [25, 36–47].

We notice that the $T_0 \equiv T(z = 0)$ data comes from the CMB blackbody radiation spectrum as estimated from the COBE collaboration [25]. The 13 low redshift ($z = 0.023 - 0.55$) $T(z)$ data are from [41], taken from multi-frequency measurements of the Sunyaev-Zeldovich (S-Z) effect toward 13 clusters (A1656, A2204, A1689, A520, A2163, A773, A2390, A1835, A697, ZW3146, RXJ1347, CL0016+16 and MS0451-0305 by order of redshift, shown on Table I). A value at high redshift comes from a damped Lyman- α (DLA) system [42, 43], from the CO excitation temperature. It was the first measurement of $T(z)$ using a molecular transition at high redshift. An additional data point is also associated to a DLA system, Q1331+170 [44] which was found by applying the inferred physical conditions to the observed C I fine structure excitation. This same process has furnished a $T(z)$ value from QSO 0013-004 [45]. Srianand *et al.* [46] reported a CMB high- z temperature from first and second fine structure levels of neutral carbon atoms in an isolated remote gas cloud. Finally, Hurier *et al.* [47] reported 18 $T(z)$ data by using thermal Sunyaev-Zeldovich (tSZ) data from the Planck satellite.

Many of these data had already been compiled by Komatsu and Kimura [29]. However, differently from the quoted authors, 5 asymmetric uncertainty data were not considered in order to avoid a result dependent over the symmetrization method choice. In addition, the 6 $T(z)$ data deduced from Planck tSZ [48] were not also used in order to prevent data duplicity.

In Fig. 1, we display all data along with the predicted $T(z)$ curves obtained from (18) for some selected values of ν_r .

With this data at hand, we performed a statistical analysis based on a χ^2 procedure, by minimizing

$$\chi^2 = \sum_{i=1}^{36} \left(\frac{T(z_i) - T_i}{\sigma_{T_i}} \right)^2 \quad (19)$$

where $T(z)$ is the modeled temperature dependence (18), T_i is the observed temperature at redshift z_i and σ_{T_i} are their uncertainties. In addition, although considering that T_0 is

TABLE I: The observed temperature–redshift relation. The 36 data points at low and intermediate redshifts are based on different observations, among them: the Sunyaev-Zeldovich effect, the rotational excitation of CO lines, the fine structure of carbon atoms, and X-ray data from galaxy clusters. By comparing with the compilation presented by Komatsu and Kimura [29] we see that 11 measured values were drawn and 18 measured values were added.

z	T (K)	Ref.
0	2.725 ± 0.002	[25]
0.023	2.72 ± 0.1	[41]
0.152	2.9 ± 0.17	[41]
0.183	2.95 ± 0.27	[41]
0.2	2.74 ± 0.28	[41]
0.202	3.36 ± 0.2	[41]
0.216	3.85 ± 0.64	[41]
0.232	3.51 ± 0.25	[41]
0.252	3.39 ± 0.26	[41]
0.282	3.22 ± 0.26	[41]
0.291	4.05 ± 0.66	[41]
0.451	3.97 ± 0.19	[41]
0.546	3.69 ± 0.37	[41]
0.55	4.59 ± 0.36	[41]
2.418	9.15 ± 0.72	[42, 43]
1.777	7.2 ± 0.8	[44]
1.973	7.9 ± 1	[45]
2.337	10 ± 4	[46]
0.037	2.888 ± 0.039	[47]
0.072	2.931 ± 0.017	[47]
0.125	3.059 ± 0.032	[47]
0.171	3.197 ± 0.03	[47]
0.22	3.288 ± 0.032	[47]
0.273	3.416 ± 0.038	[47]
0.322	3.562 ± 0.05	[47]
0.377	3.717 ± 0.063	[47]
0.428	3.971 ± 0.071	[47]
0.471	3.943 ± 0.112	[47]
0.525	4.38 ± 0.119	[47]
0.565	4.075 ± 0.156	[47]
0.618	4.404 ± 0.194	[47]
0.676	4.779 ± 0.278	[47]
0.718	4.933 ± 0.371	[47]
0.777	4.515 ± 0.621	[47]
0.87	5.356 ± 0.617	[47]
0.972	5.813 ± 1.025	[47]

nicely determined by the COBE analysis, we choose to leave it as a free parameter, in order

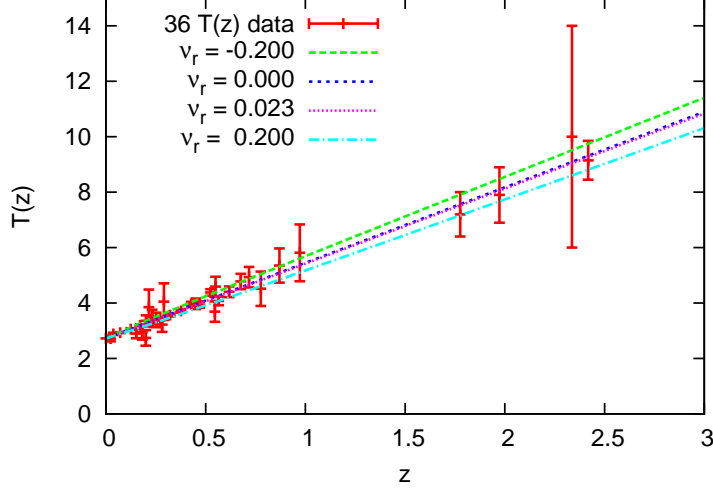


FIG. 1: $T(z)$ data from many references, as listed on Table I and the prediction of some models as depicted in the figure. Note that the blue dotted line ($\nu_r = 0$) corresponds to the standard Λ CDM.

to check the consistence of the model.

In Fig. 2a, we present the confidence contours on the (T_0, ν_r) plane. From the figure we see that the (T_0, ν_r) plane is nicely constrained by this analysis. The results obtained from the joint analysis are: $\nu_r = 0.023^{+0.041+0.066+0.091}_{-0.042-0.069-0.095}$ and $T_0 = 2.7252 \pm 0.0030 \pm 0.0050 \pm 0.068$ at 68.3%, 95.4% and 99.7%, respectively. We also have found the minimal value, $\chi^2_{min} = 28.219$, and the reduced one $\chi^2_{red} = 0.830$.

Next, in order to focus on the ν_r constraint, we marginalized the likelihood $\mathcal{L} = Ne^{-\frac{\chi^2}{2}}$, where N is a normalization constant, over T_0 , which can be done analytically, as $T(z)$ is linearly dependent over T_0 . In fact, we find a “marginalized” $\tilde{\chi}^2$ from the marginalized likelihood $\tilde{\mathcal{L}}(\nu_r) \equiv \int_{-\infty}^{\infty} \mathcal{L}(\nu_r, T_0) dT_0$ as

$$\tilde{\chi}^2 = -2 \ln \left(\frac{\tilde{\mathcal{L}}}{N} \right) = S_{TT} - \frac{S_{fT}^2}{S_{ff}} \quad (20)$$

where we have defined $f(z) \equiv \frac{T(z)}{T_0}$ from Eq. (18) and

$$S_{ff} \equiv \sum_{i=1}^{36} \frac{f(z_i)^2}{\sigma_{Ti}^2}, \quad S_{fT} \equiv \sum_{i=1}^{36} \frac{f(z_i)T_i}{\sigma_{Ti}^2}, \quad S_{TT} \equiv \sum_{i=1}^{36} \frac{T_i^2}{\sigma_{Ti}^2}. \quad (21)$$

In Fig. 2b we present the likelihood of the production rate ν_r . We have found, from this analysis, $\nu_r = 0.023^{+0.027+0.054+0.080}_{-0.027-0.055-0.083}$, at 68.3%, 95.4% and 99.7%, respectively. Note that for the central value of ν_r the α_r parameter modulating the creation rate, $\alpha_r = \nu_r \Omega_{r0} \sim 10^{-7}$.

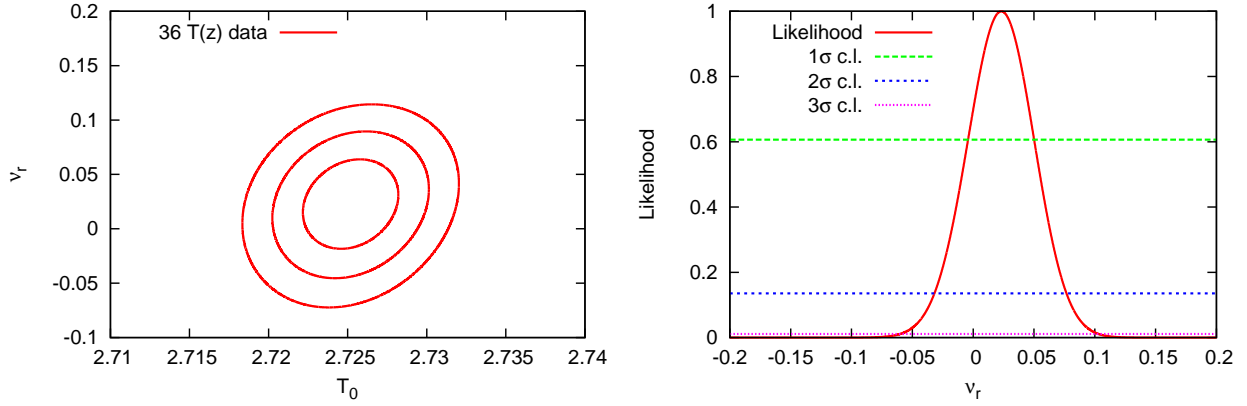


FIG. 2: **a)** Confidence contours on the (T_0, ν_r) plane for the CCDM cosmology as inferred from 36 $T(z)$ data at low and intermediate redshifts (see Table I and Fig. 1). The best fit values are located at $(T_0, \nu_r) \equiv (2.7252, 0.023)$. **b)** The likelihood of the production rate free parameter ν_r in the CCDM cosmology. It is peaked for a positive value (creation) and is constrained by $\nu_r = 0.023 \pm 0.027$ at 1σ confidence level. Note also that for this data sample the Λ CDM model ($\nu_r = 0$) is also marginally compatible at 1σ .

V. CONCLUSION

The standard Λ CDM model can be mimicked at the background and perturbative levels (linear and non-linear) by a class of gravitationally induced particle production cosmology dubbed CCDM model.

In this paper, we have discussed a new probe to CCDM cosmology provided by the so-called thermal sector, namely: the temperature-redshift $T(z)$ -law. The predictions of CCDM cosmology were analyzed and we have demonstrated that this reduction of the dark sector with photon production passes this simple temperature test. The comparison with the Λ CDM results was also performed (see Figures 1 and 2). The nonlinear $T(z)$ expression derived in this context differs significantly from the linear law predicted by the Λ CDM model. The more general expression is modulated by a phenomenological free parameter (ν_r) associated to the gravitationally induced photon production rate.

In order to constrain ν_r we have carried out a statistical analysis based on 36 recent measurements of $T(z)$ at low and moderate redshifts. The likelihood of the production rate in CCDM cosmologies constrains its value to $\nu_r = 0.023 \pm 0.027$ (1σ confidence level) thereby

showing that Λ CDM ($\nu_r = 0$) is also marginally compatible (at 1σ) with the adopted data sample (see Fig. 2b). Due to the “adiabatic” photon production, our results suggest (for redshifts of the order of a few) that the CMB temperature in the context of CCDM models may be slightly lower than the value predicted by the Λ CDM cosmology. However, accurate data at low and moderate z are needed in order to have a more definite conclusion.

Finally, by taking into account the results obtained here and the fact that measurements of H_0 at low and high redshifts are now endowed with a 3σ -level discrepancy (Supernovae-CMB tension [24]), we believe that probes in the thermal sector which are responsible for the success of the fitting Λ CDM model should be carefully reinvestigated. Potentially, even conserved quintessence models can be ruled out by cosmic probes in the thermal sector, mainly when confronted with a model allowing a mild gravitationally induced production of photons. The present results clearly reinforces the idea that the present observed Λ CDM description may be only an effective CCDM cosmology.

Appendix A: Kinetic Theory and Temperature Law

In this appendix, we use the extended Boltzmann equation for gravitationally particle production, proposed in previous works [34, 49], to get the temperature evolution of the relic radiation.

In a multi-fluid approach, each component has its own equation with its corresponding production rate. The extended Boltzmann equation describing this gravitational, non-collisional (each component evolves freely from the others), process is

$$\frac{\partial f_i}{\partial t} - H \left(1 - \frac{\Gamma_i}{3H} \right) p \frac{\partial f_i}{\partial p} = 0, \quad (\text{A1})$$

where f_i and Γ_i are, respectively, the distribution function and the production rate for the i -th component.

For a relativistic quantum gas, the distribution function is [50]

$$f = \frac{1}{e^{-\Theta + \beta E} + \epsilon}, \quad (\text{A2})$$

where Θ is the relativistic chemical potential, $\beta = 1/T$, where T is the temperature, and $\epsilon = \pm 1$ counts for different quantum statistics.

In the case of a relativistic bosonic gas with creation rate Γ_r (CMB radiation), by inserting (A2) into (A1) we obtain (see also [34]):

$$\frac{\dot{\Theta}}{\dot{\beta}} = E \left[1 - H \frac{\beta}{\dot{\beta}} \left(1 - \frac{\Gamma_r}{3H} \right) \right], \quad (\text{A3})$$

which has the solution $\dot{\Theta} = 0$ and

$$\frac{\dot{T}_r}{T_r} = -\frac{\dot{a}}{a} + \frac{\Gamma_r}{3}, \quad (\text{A4})$$

where a is the scale factor and Γ_r stands for the radiation production rate. The above expression is the same as Eq. (15) which was deduced based on the thermodynamic approach. In the limit $\frac{\Gamma_r}{3H} \ll 1$, equations (A3) and (A4) are reduced to the usual ones (see equations (3.70) and (3.71) in [51]).

Acknowledgments: The authors are partially supported by CNPq, FAPESP and CAPES (LLAMA project, INCT-A and PROCAD2013 projects).

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